

Cosmological evolution in DHOST theories

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In the context of Degenerate Higher-Order Scalar-Tensor (DHOST) theories, we study cosmological solutions and their stability properties. In particular, we explicitly illustrate the crucial role of degeneracy by showing how the higher order homogeneous equations in the physical frame (where matter is minimally coupled) can be recast in a system of equations that do not involve higher order derivatives. We study the fixed points of the dynamics, finding the conditions for having a de Sitter attractor at late times. Then we consider the coupling to matter field (described for convenience by a k-essence Lagrangian) and find the conditions to avoid gradient and ghost instabilities at linear order in cosmological perturbations, extending previous work. Finally, we apply these results to a simple subclass of DHOST theories, showing that de Sitter attractor conditions, no ghost and no gradient instabilities conditions (both in the self-accelerating era and in the matter dominated era) can be compatible.

I. INTRODUCTION

Since the discovery of the accelerated expansion of our universe, many models of dark energy or modified gravity have been proposed to account for this unexpected observation (see [1–4] for reviews). Among these models, scalar-tensor theories of gravity have played a prominent role as they simply add a scalar degree of freedom to the usual tensor modes of general relativity. In order to have a general understanding of the impact of scalar-tensor theories on cosmology and on astrophysics, it is convenient to resort to a unified approach that can describe as many models as possible with the same formalism. The most general framework that has been developed so far is that of Degenerate Higher-Order Scalar-Tensor (DHOST) theories [5, 6] (see also [7, 8]), which extends the family of Horndeski [9] (also known as generalized galileons [10–12]) and Beyond Horndeski (or GLPV) theories [13, 14].

DHOST theories allow for the presence of second-order derivatives of the scalar field ϕ , i.e. of $\nabla_\mu \nabla_\nu \phi$ in the Lagrangian, as in Horndeski theories. However, in contrast to the latter, which are restricted to Lagrangians leading to second-order Euler-Lagrange equations (for both the metric and the scalar field), DHOST theories allow for higher-order Euler-Lagrange equations but are required to contain only one scalar degree of freedom in order to avoid Ostrogradski instabilities, associated with an extra degree of freedom that often appears in systems with higher order time derivatives. The possibility of having higher order Euler-Lagrange equations without an extra degree of freedom was illustrated by disformal transformations of the Einstein-Hilbert Lagrangians [15] and by Beyond Horndeski (or GLPV) Lagrangians [13, 14].

It was later realized that the crucial property shared by these models is the degeneracy of their Lagrangian, which guarantees the absence of a potentially disastrous extra degree of freedom [5].

The absence of an extra degree of freedom was confirmed, for Beyond Horndeski theories, by their relation to Horndeski theories via field redefinition [14, 16] as well as a Hamiltonian analysis for a particular quadratic case [17], and for quadratic DHOST theories, by a general Hamiltonian analysis [18]¹. In the case of Beyond Horndeski theories, let us stress that the degeneracy restricts the possible combinations of quadratic and cubic terms [5, 16], meaning that a special tuning between the free functions of Beyond Horndeski theories is required [6].

The purpose of this work is to explore the cosmology of DHOST theories, first at the level of background evolution and then for linear perturbations. Some preliminary investigations were conducted in [22] for a special class of DHOST theories (see [23, 24] for a similar analysis in Horndeski theories and [25] in Beyond Horndeski theories). In the present work, we illustrate explicitly the crucial role of degeneracy by showing how the higher order homogeneous equations in the physical frame (where matter is minimally coupled) can be recast in a system of equations that do not involve higher order derivatives. This second system corresponds to another frame, which we call Horndeski frame, where matter is non-minimally coupled. The transition from one frame to the other is given by a disformal transformation of the metric, similarly to the conformal transformation that relates the physical (or Jordan) and Einstein frames in traditional scalar-tensor theories.

On using the homogeneous cosmological evolution equations, we study the existence of self-accelerating solutions, and then compute the conditions for these fixed points to be attractors. We also construct scaling solutions which interpolate between the matter and the accelerating eras. We then study linear cosmological perturbations about these different solutions. Using the effective theory of dark energy perturbations applied to DHOST theory [26] (based on and extending the previous works [27–29]), we obtain the Lagrangian governing the linear degrees of freedom, thus identifying the conditions that guarantee the linear stability of the system. Finally, we apply this general analysis to a simple illustrative example of a DHOST theory.

The outline of the paper is the following. In the next section, we introduce DHOST theories, focusing on the phenomenologically viable subclass (quadratic theories with stable linear perturbations), which contains Horndeski and Beyond Horndeski theories, and we describe the homogeneous cosmological dynamics in terms of the scale factor of the physical frame or that of the Horndeski frame. Then, at the end of section II, we identify self-accelerating solutions and investigate whether or not they are attractors with respect to the homogeneous cosmological evolution. Linear cosmological perturbations are studied in section III: we consider first the pure scalar-tensor action and then include a matter action consisting of a k-essence type scalar field. Section IV is devoted to the application of our results to a simple and illustrative family of models. We give a conclusion in the final section.

II. DHOST THEORIES AND HOMOGENEOUS COSMOLOGICAL EQUATIONS

We start with the general quadratic DHOST action [5, 7, 8]

$$S[g_{\mu\nu}, \phi] = \int d^4x \sqrt{-g} \left[F_0(\phi, X) + F_1(\phi, X) \square \phi + F_2(\phi, X) R + \sum_{I=1}^5 A_I(\phi, X) L_I^{(2)} \right], \quad (2.1)$$

where $g_{\mu\nu}$ is the metric to which matter is minimally coupled, i.e. $g_{\mu\nu}$ corresponds to the physical frame (or Jordan frame) metric. Furthermore, $X \equiv g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$, and the last term in (2.1) contains

¹ For a systematic analysis of higher derivative theories and the conditions required to avoid Ostrogradski instabilities, see [19, 20] for classical mechanics and [21] for field theories.

all five possible Lagrangians $L_A^{(2)}$, quadratic in second derivatives of the field. They are given by

$$\begin{aligned} L_1^{(2)} &\equiv \phi^{\mu\nu}\phi_{\mu\nu}, & L_2^{(2)} &\equiv (\phi_\nu{}^\nu)^2, & L_3^{(2)} &\equiv \phi_\nu{}^\nu\phi^\rho\phi_{\rho\sigma}\phi^\sigma, \\ L_4^{(2)} &\equiv \phi^\mu\phi_{\mu\nu}\phi^{\nu\rho}\phi_\rho, & L_5^{(2)} &\equiv (\phi^\rho\phi_{\rho\sigma}\phi^\sigma)^2, \end{aligned} \quad (2.2)$$

where $\phi_{\mu\nu} = \nabla_\nu\nabla_\mu\phi$, and $\phi_\mu = \nabla_\mu\phi$.

A. Degenerate and isokinetic theories

The functions F_2 and A_I satisfy three degeneracy conditions, as given in [5], such that the DHOST action (2.1) propagates only one scalar degree of freedom, as well as two tensor modes. Here we focus on the class of DHOST theories that includes Horndeski and Beyond Horndeski theories [7, 8], as it is the only phenomenologically viable one with real propagation speeds (with no gradient instabilities at linear order) [26, 30]. This class has been also shown to be stable under quantum corrections [31].

Due to the three degeneracy conditions, the action is described in term of five independent functions: F_0 , F_1 , F_2 and two among the remaining five functions A_I . We will take these to be A_1 and A_3 whereas the others are given by

$$A_2 = -A_1 \neq -F_2/X, \quad (2.3)$$

$$A_4 = \frac{1}{8(F_2 - A_1X)^2} [4F_2(3(A_1 - 2F_{2X})^2 - 2A_3F_2) - A_3X^2(16A_1F_{2X} + A_3F_2) + 4X(3A_1A_3F_2 + 16A_1^2F_{2X} - 16A_1F_{2X}^2 - 4A_1^3 + 2A_3F_2F_{2X})], \quad (2.4)$$

$$A_5 = \frac{1}{8(F_2 - A_1X)^2}(2A_1 - A_3X - 4F_{2X})[A_1(2A_1 + 3A_3X - 4F_{2X}) - 4A_3F_2]. \quad (2.5)$$

As can be seen directly from the 3+1 decomposition of the DHOST action given in [5], the speed of propagation of gravitational waves, c_g , is equal to the speed of light $c = 1$ only when $A_1 = 0$, in which case the weights of the kinetic term $K_{ij}K^{ij}$ and of the 3 dimensional scalar curvature ${}^{(3)}R$ are the same. The constraints inferred from the equality of the speed of light and that of gravitational waves, following the observation of the neutron star merger GW170817, have been discussed in [32–38]. We will refer to the corresponding subclass as “isokinetic” theories. However, for the purpose of generality, we do not impose this constraint for the moment.

B. Cosmological action

Now consider a homogeneous and isotropic universe with a Friedmann-Lemaître-Robertson-Walker (FLRW) spatially flat metric

$$ds^2 = -N^2dt^2 + a^2\delta_{ij}dx^idx^j, \quad (2.6)$$

where the lapse N and the scale factor a depend only on time, as does the scalar field ϕ . Inserting this metric in the action (2.1) and taking into account the degeneracy conditions, (2.1) yields the homogeneous action

$$\begin{aligned} S_{\text{hom}} = \int dt Na^3 \left\{ -6(F_2 - XA_1) \left[\frac{\dot{a}}{Na} - \mathcal{V} \frac{\dot{\phi}}{N^2} \frac{d}{dt} \left(\frac{\dot{\phi}}{N} \right) \right]^2 \right. \\ \left. - 3(F_1 + 2F_{2\phi}) \frac{\dot{a}\dot{\phi}}{N^2a} - F_1 \frac{1}{N} \frac{d}{dt} \left(\frac{\dot{\phi}}{N} \right) + F_0 \right\}, \end{aligned} \quad (2.7)$$

where $F_{2\phi} \equiv \partial_\phi F_2$, all the free functions now depend on $\phi(t)$ and $X \equiv -\dot{\phi}^2/N^2$, and \mathcal{V} is given by

$$\mathcal{V} \equiv \frac{4F_{2X} + X A_3 - 2A_1}{4(F_2 - X A_1)}. \quad (2.8)$$

Note that, as a result of the degeneracy, the terms quadratic in \dot{a} and $\ddot{\phi}$ combine into a square term in the first line of (2.7). This motivates the definition of a new scale factor, b , which absorbs the second time derivative of the scalar field,

$$\frac{\dot{b}}{b} = \frac{\dot{a}}{a} - \mathcal{V} \frac{\dot{\phi}}{N} \frac{d}{dt} \left(\frac{\dot{\phi}}{N} \right) + \dots, \quad (2.9)$$

where the dots denote terms up to first order in time derivatives of ϕ . Letting

$$a = e^{\lambda(X, \phi)} b \quad \Rightarrow \quad \frac{\dot{a}}{a} = \frac{\dot{b}}{b} + \lambda_X \dot{X} + \lambda_\phi \dot{\phi}, \quad (2.10)$$

it follows from (2.9) that

$$\lambda_X = -\frac{1}{2} \mathcal{V} = -\frac{4F_{2X} + X A_3 - 2A_1}{8(F_2 - X A_1)}. \quad (2.11)$$

Re-expressing the cosmological action (2.7) in terms of the new scale factor b gives

$$S_{\text{hom}} = \int dt N b^3 \left[\hat{F}_2(X, \phi) \frac{\dot{b}^2}{N^2 b^2} + \hat{F}_1(X, \phi) \frac{\dot{b} \dot{\phi}}{N^2 b} + \hat{F}_0(X, \phi) + \hat{G}_1(X, \phi) \frac{1}{N} \frac{d}{dt} \left(\frac{\dot{\phi}}{N} \right) \right] \quad (2.12)$$

where it is straightforward to see that the functions \hat{F}_I as well as \hat{G}_1 are given by

$$\begin{aligned} \hat{F}_2 &\equiv -6e^{3\lambda}(F_2 - X A_1), \\ \hat{F}_1 &\equiv -3e^{3\lambda} [F_1 + 2F_{2\phi} + 4(F_2 - X A_1)\lambda_\phi], \end{aligned} \quad (2.13)$$

$$\begin{aligned} \hat{G}_1 &\equiv -e^{3\lambda} [F_1 + 6X(F_1 + 2F_{2\phi})\lambda_X], \\ \hat{F}_0 &\equiv e^{3\lambda} [F_0 + 3X(F_1 + 2F_{2\phi})\lambda_\phi + 6X(F_2 - X A_1)\lambda_\phi^2]. \end{aligned} \quad (2.14)$$

In general λ is defined implicitly by the differential equation (2.11). In the special isokinetic case $A_1 = 0$, (2.11) reduces to

$$\lambda_X = -\frac{4F_{2X} + X A_3}{8F_2}. \quad (2.15)$$

In some cases, the integration can be done explicitly. In particular, when $X A_3$ is proportional to F_{2X} , namely $X A_3 = -(4 + 8\mu)F_{2X}$ where μ is a constant, (2.15) gives $e^\lambda = \left(\frac{F_2}{M^2}\right)^\mu$, where M^2 is an arbitrary (possibly ϕ -dependent) function. We will consider this example in further detail below (see section IV).

From now on, we focus on shift symmetric theories in which all the functions appearing in the action depend only on X , and also $\lambda = \lambda(X)$. The reason is that, below, we will be interested in fixed points of the equations of motion. In non-shift symmetric theories, the fixed points are necessarily characterized by $\dot{\phi} = 0 = X$. In that case $\phi = \text{constant}$, and self-acceleration can only be generated by a potential for ϕ . Here we are interested in situations in which dark energy is generated by the dynamics of the field itself.

C. Equations of motion

Starting from the action (2.12), we now write down the equations of motion. In the “Horndeski frame” with scale factor b , matter is not coupled minimally to the metric². How this non-minimal coupling appears in the Friedmann equations is explained in the Appendix.

In terms of the original functions F_I and A_I , the Friedmann equations are given respectively by

$$6 \left[F_2 + (2F_{2X} + 6\lambda_X F_2 - 3A_1)X - 2(A_{1X} + 3\lambda_X A_1)X^2 \right] H_b^2 + 6F_{1X}XH_b\dot{\phi} - 2(F_{0X} + 3\lambda_X F_0)X + F_0 = (1 + 6w_m X \lambda_X) \rho_m, \quad (2.16)$$

and

$$2(F_2 - XA_1)(2\dot{H}_b + 3H_b^2) + F_0 + 2F_{1X}X\ddot{\phi} + 4[F_{2X} + 3\lambda_X F_2 - A_1 - X(A_{1X} + 3\lambda_X A_1)]\dot{X}H_b = -P_m, \quad (2.17)$$

where λ_X is given in (2.11), and ρ_m and P_m are respectively the energy density and pressure defined in the physical frame. In the following we assume an equation of state $P_m = w_m \rho_m$ with w_m constant, which applies to the radiation dominated and matter dominated eras respectively. Notice that this second equation contains $\ddot{\phi}$ and \dot{H}_b , and that A_3 only appears in the function λ_X .

In the isokinetic case $A_1 = 0$, which we now consider for the remainder of this paper, the above equations simplify

$$\mathcal{E}_1 \equiv 6[F_2 + (2F_{2X} + 6\lambda_X F_2)X]H_b^2 + 6F_{1X}XH_b\dot{\phi} - 2(F_{0X} + 3\lambda_X F_0)X + F_0 = (1 + 6w_m X \lambda_X) \rho_m, \quad (2.18)$$

and

$$\mathcal{E}_2 \equiv 2F_2(2\dot{H}_b + 3H_b^2) + F_0 + 2F_{1X}X\ddot{\phi} + 4(F_{2X} + 3\lambda_X F_2)\dot{X}H_b = -P_m, \quad (2.19)$$

In order to obtain the dynamical equation for ϕ – that involves only $\ddot{\phi}$ but not \dot{H}_b – we first consider the combination

$$w_m \mathcal{E}_1 + (1 + 6w_m X \lambda_X) \mathcal{E}_2 = 0,$$

where the right hand side vanishes since $P_m = w_m \rho_m$. This equation is linear in \dot{H}_b and yields

$$\begin{aligned} \dot{H}_b = & -\frac{1}{4F_2(1 + 6w_m X \lambda_X)} \left\{ (1 + w_m)F_0 - 2w_m X F_{0X} \right. \\ & + 6H_b^2 [2w_m X F_{2X} + (1 + w_m + 12w_m X \lambda_X)F_2] + 2X F_{1X}(1 + 6w_m X \lambda_X)\ddot{\phi} \\ & \left. + 2H_b [3w_m X F_{1X} - 4(F_{2X} + 3F_2 \lambda_X)(1 + 6w_m X \lambda_X)\dot{\phi}] \dot{\phi} \right\}. \end{aligned} \quad (2.20)$$

We then substitute the above expression for \dot{H}_b into the combination

$$(1 + 6w_m X \lambda_X) \left[\dot{\mathcal{E}}_1 + 3 \left(H_b - 2\lambda_X \dot{\phi} \ddot{\phi} \right) (\mathcal{E}_1 - (1 + 6w_m X \lambda_X) \mathcal{E}_2) \right] - 6w_m (X \lambda_X) \dot{\mathcal{E}}_1 = 0, \quad (2.21)$$

which vanishes as a consequence of the conservation equation $\dot{\rho}_m + 3H(\rho_m + P_m) = 0$. This yields the equation of motion for the scalar field, which we do not write explicitly as it is quite involved.

² If one describes matter by a k -essence like action $S_{matter} = \int d^4x \sqrt{-g} P(\sigma, \partial_\mu \sigma \partial^\mu \sigma)$ in the DHOST frame with scale factor a , then in the “Horndeski” frame, and in the case of the homogenous background, matter becomes conformally coupled to b , $S_{matter} = \int dt b^3 e^{3\lambda} P(\sigma, -\dot{\sigma}^2)$.

D. Self-accelerating attractors

The aim of this part is to determine the conditions under which it is possible to have self-accelerating cosmological expansion in DHOST theories.

Self-accelerating de Sitter solutions can be easily identified by considering the equations of motion in the absence of matter, and imposing $\ddot{\phi} = 0$ and $\dot{H}_b = 0$. Equation (2.19) directly yields

$$H_b = \sqrt{-\frac{F_0}{6F_2}} \equiv H_{dS}, \quad (2.22)$$

which requires $F_0 F_2 < 0$. Substituting in equation (2.18) gives

$$2[(F_0 F_2)_X + 6\lambda_X F_0 F_2]^2 - 3F_0 F_2 F_{1X}^2 X = 0, \quad (2.23)$$

which is an equation for the constant $X \equiv X_{dS}$. We will study in section IV the simple case in which $F_1 = 0$, and then X_{dS} satisfies the equation

$$(F_0 F_2)_X + 6\lambda_X F_0 F_2 = 0. \quad (2.24)$$

In order to find the conditions under which the solution defined by (2.22) and (2.23) is an attractor, we introduce the perturbations

$$x \equiv X - X_{dS}, \quad h \equiv H_b - H_{dS}.$$

Expanding (2.18) to linear order gives h in terms of x ; and then substituting this expression into (2.19) gives a first order equation for the perturbation $x(t)$,

$$\mathcal{A} \dot{x} + \mathcal{B} x = 0,$$

with \mathcal{A} and \mathcal{B} given by

$$\begin{aligned} \mathcal{A} \equiv & -2F_2^2 [2XF_{0XX} + F_{0X}(6X\lambda_X + 1) + 6F_0(2X\lambda_{XX} - 6X\lambda_X^2 + \lambda_X)] \\ & + F_2 \{2F_0[F_{2X}(18X\lambda_X - 1) - 2XF_{2XX}] + 3X^2 F_{1X}^2\} + 8XF_0 F_{2X}^2 \\ & + 2\sqrt{6XF_0 F_2} \{F_2[XF_{1XX} + F_{1X}(1 - 6X\lambda_X)] - 2XF_{1X} F_{2X}\}, \end{aligned} \quad (2.25)$$

$$\begin{aligned} \mathcal{B} \equiv & -3\sqrt{-X} \{F_{1X}[XF_2 F_{0X} + F_0(3F_2 - XF_{2X})] + 2XF_0 F_2 F_{1XX}\} \\ & - 2\sqrt{-\frac{6F_0}{F_2}} \{F_2[(XF_{0X} + F_0)F_{2X} + XF_0 F_{2XX}] \\ & + F_2^2[XF_{0XX} + F_{0X}(6X\lambda_X + 1) + 6F_0(X\lambda_{XX} + \lambda_X)] - XF_0 F_{2X}^2\}, \end{aligned} \quad (2.26)$$

where X is evaluated at the fixed point X_{dS} . The condition for the self-accelerating universe to be an attractor is

$$\mathcal{A}\mathcal{B} > 0. \quad (2.27)$$

Notice that this condition is of course frame independent; had we worked from the start with a and H (rather than b and H_b) the result would have been the same since, from (2.10), at the fixed point $\dot{X} = 0$ so that $H = H_b$.

We will consider this condition below (see section IV), in conjunction with the stability conditions coming from the study of linear cosmological perturbations to which we now turn.

III. STABILITY OF LINEAR COSMOLOGICAL PERTURBATIONS

In this section, we derive the second-order action for cosmological perturbations, and study it in the de Sitter phase (2.22), as well as in the case of a matter dominated universe. We begin by considering the case of an empty universe.

A. Quadratic action in a cosmological background

Contrary to the analysis of section IID, here we work in the physical frame with scale factor a . Linear cosmological perturbations around FLRW background have been studied for DHOST theories in [26, 30]. The quadratic action for the scalar perturbation $\zeta(t, x)$ (in the unitary gauge, and ignoring any matter contribution) was found to be [26]

$$S_{\text{quad}}[\zeta] = \int d^3x dt a^3 \frac{M^2}{2} \left[A_\zeta \dot{\zeta}^2 + B_\zeta \frac{(\partial_i \zeta)^2}{a^2} \right],$$

where

$$A_\zeta = \frac{1}{(1 + \alpha_B - \dot{\beta}_1/H)^2} \left[\alpha_K + 6\alpha_B^2 - \frac{6}{a^3 H^2 M^2} \frac{d}{dt} (a^3 H M^2 \alpha_B \beta_1) \right], \quad (3.1)$$

$$B_\zeta = 2(1 + \alpha_T) - \frac{2}{a M^2} \frac{d}{dt} \left[\frac{a M^2 (1 + \alpha_H + \beta_1(1 + \alpha_T))}{H(1 + \alpha_B) - \dot{\beta}_1} \right], \quad (3.2)$$

with $\alpha_T = 0$ (since we focus here on the isokinetic case), and the parameters α_K , α_B , α_H and β_1 are given in terms of the free functions in (2.1), by

$$M^2 = 2F_2, \quad \alpha_H = -\frac{2XF_{2X}}{F_2}, \quad \beta_1 = \frac{X(4F_{2X} + XA_3)}{4F_2}, \quad (3.3)$$

$$\alpha_B = -\frac{X(4HF_{2X} + 3HXA_3 - 2\sqrt{-X}F_{1X})}{4HF_2}, \quad (3.4)$$

$$\begin{aligned} \alpha_K = \frac{X}{2H^2 F_2} \Big\{ & 2 \left[3X^2 (\dot{H} + 3H^2) A_{3X} + 2X (F_{0XX} + 6(\dot{H} + 2H^2) F_{2XX}) \right. \\ & + F_{0X} + 6H\sqrt{-X} (XF_{1XX} - F_{1X}) + 6(3\dot{H} + 2H^2) F_{2X} \Big] \\ & \left. + 3XA_3 (5\dot{H} + 9H^2) \right\}. \end{aligned} \quad (3.5)$$

The stability conditions (no ghost and no gradient instabilities) for linear cosmological perturbations are

$$A_\zeta > 0, \quad B_\zeta < 0. \quad (3.6)$$

At the de-Sitter fixed point (2.22), the expressions for A_ζ and B_ζ are given by

$$A_\zeta = \left\{ 18X \left[2F_2^2 (2XF_{0XX} + F_{0X} + 6F_0 (2X\lambda_{XX} - 12X\lambda_X^2 + \lambda_X)) \right. \right. \\ \left. + F_2 (2F_0F_{2X} + 4XF_0 (F_{2XX} - 12F_{2X}\lambda_X) - 3X^2F_{1X}^2) - 8XF_0F_{2X}^2 \right] \\ \left. + 36\sqrt{6}X\sqrt{XF_0F_2} [2XF_{1X}F_{2X} + F_2 (F_{1X} (9X\lambda_X - 1) - XF_{1XX})] \right\} / (F_2D^2), \quad (3.7)$$

$$B_\zeta = \left\{ X6\sqrt{6}X\sqrt{XF_0F_2}F_{1X} [6XF_{2X} + F_2 (14X\lambda_X + 1)] \right. \\ \left. - 6X [F_2 (80XF_0F_{2X}\lambda_X + 8F_0F_{2X} + 3X^2F_{1X}^2) \right. \\ \left. + 16XF_0F_{2X}^2 + 16F_0F_2^2\lambda_X (6X\lambda_X + 1)] \right\} / (F_2D^2), \quad (3.8)$$

where X is evaluated at the fixed point X_{dS} given by the solution of (2.23), and

$$D = X \left(2\sqrt{6}\sqrt{-\frac{F_0}{F_2}}F_{2X} + 3\sqrt{-X}F_{1X} \right) + \sqrt{6}\sqrt{-F_0F_2} (6X\lambda_X + 1). \quad (3.9)$$

We will illustrate these stability conditions, and their relation to (2.27), with a specific example in section IV.

B. Including matter

We now extend our analysis and consider the case in which matter is present. This is interesting in order to see whether these theories could reproduce a history of the universe with a matter dominated era before the self-accelerating one. We will show that this is indeed the case with the simple example of section IV.

We describe matter as a scalar field $\sigma(t, x)$, with a k-essence type action

$$S_m = \int d^4x \sqrt{-g} P(Y), \quad Y \equiv g^{\mu\nu} \partial_\mu \sigma \partial_\nu \sigma,$$

which should be added to (2.1). To make contact with the perfect fluid description of matter, we identify the matter density ρ_m , the equation of state $P_m = w_m \rho_m$ and the sound speed c_m

$$w_m = -\frac{P}{P + 2\dot{\sigma}_0^2 P_Y}, \quad c_m^2 = \frac{P_Y}{P_Y - 2\dot{\sigma}_0^2 P_{YY}}, \quad (3.10)$$

where $\sigma_0(t)$ denotes the value of σ on the cosmological background.

Following the same analysis as in [26] and after a long calculation, the quadratic action for the perturbations reduces to

$$S_{\text{quad}}[v] = \int d^3x dt a^3 \frac{M^2}{2} \left[\dot{v}^\top \mathbf{K} \dot{v} + v^\top \mathbf{D} \dot{v} + \frac{1}{a^2} \partial_i v^\top \mathbf{L} \partial_i v + v^\top \mathbf{M} v \right], \quad (3.11)$$

where $v^\top \equiv (\zeta, \delta\sigma)$ and

$$\mathbf{K} = \begin{pmatrix} A_\zeta + \frac{\rho_m(1+w_m)}{M^2 c_m^2 (H(1+\alpha_B) - \dot{\beta}_1)^2} & \frac{\rho_m(1+w_m)(3c_m^2\beta_1 - 1)}{M^2 c_m^2 (H(1+\alpha_B) - \dot{\beta}_1)\dot{\sigma}_0} \\ \frac{\rho_m(1+w_m)(3c_m^2\beta_1 - 1)}{M^2 c_m^2 (H(1+\alpha_B) - \dot{\beta}_1)\dot{\sigma}_0} & \frac{\rho_m(1+w_m)}{M^2 c_m^2 \dot{\sigma}_0^2} \end{pmatrix}, \quad \mathbf{D} = \begin{pmatrix} 0 & d \\ d & 0 \end{pmatrix}, \quad (3.12)$$

$$\mathbf{L} = \begin{pmatrix} B_\zeta & \frac{\rho_m(1+w_m)(1+\alpha_H + (1+\alpha_T)\beta_1)}{M^2 (H(1+\alpha_B) - \dot{\beta}_1)\dot{\sigma}_0} \\ \frac{\rho_m(1+w_m)(1+\alpha_H + (1+\alpha_T)\beta_1)}{M^2 (H(1+\alpha_B) - \dot{\beta}_1)\dot{\sigma}_0} & -\frac{\rho_m(1+w_m)}{M^2 \dot{\sigma}_0^2} \end{pmatrix}, \quad \mathbf{M} = \begin{pmatrix} 0 & q \\ q & m \end{pmatrix}. \quad (3.13)$$

Notice that the explicit forms of m, q and d (which are very involved) are not relevant for our purposes, and A_ζ and B_ζ are given in (3.1) and (3.2) respectively.

The stability conditions, in order to avoid ghost and gradient instabilities, translate, respectively, into the requirement that the eigenvalues of \mathbf{K} should be positive and those of \mathbf{L} should be negative. Let us study these conditions in a matter dominated era where the radiation contribution to the matter content is supposed to be negligible.

For that purpose, we first denote these eigenvalues by

$$\text{Eigen}(\mathbf{K}) = \{\lambda_{\mathbf{K}_1}, \lambda_{\mathbf{K}_2}\}, \quad \text{Eigen}(\mathbf{L}) = \{\lambda_{\mathbf{L}_+}, \lambda_{\mathbf{L}_-}\}.$$

In the matter dominated era, we assume $c_m \simeq w_m \ll 1$, so that the leading contribution in an expansion in w_m gives

$$\lambda_{\mathbf{K}_1} \simeq \frac{A_\zeta M^2 \left(H(1+\alpha_B) - \dot{\beta}_1 \right)^2 + 6\rho_m\beta_1}{M^2 \left[\left(H(1+\alpha_B) - \dot{\beta}_1 \right)^2 + \dot{\sigma}_0^2 \right]}, \quad \lambda_{\mathbf{K}_2} \simeq \frac{\rho_m}{M^2 w_m^2} \left[\frac{1}{\left(H(1+\alpha_B) - \dot{\beta}_1 \right)^2} + \frac{1}{\dot{\sigma}_0^2} \right], \quad (3.14)$$

$$\lambda_{\mathbf{L}_\pm} \simeq \frac{B_\zeta}{2} - \frac{1}{2M^2 \dot{\sigma}_0^2} \left[\rho_m \pm \sqrt{\frac{4\rho_m^2 (1+\alpha_H + (1+\alpha_T)\beta_1)^2 \dot{\sigma}_0^2}{\left(H(1+\alpha_B) - \dot{\beta}_1 \right)^2} + (\rho_m + B_\zeta M^2 \dot{\sigma}_0^2)^2} \right]. \quad (3.15)$$

Concerning the eigenvalues of \mathbf{K} , one finds that $\lambda_{\mathbf{K}_2}$ in (3.14) is always positive whereas $\lambda_{\mathbf{K}_1}$ is positive only when its numerator is positive. The analysis of the eigenvalues of \mathbf{L} is slightly more subtle because it involves the background evolution of σ .

The background field equation of σ is $\ddot{\sigma}_0 + 3c_m^2 H \dot{\sigma}_0 = 0$ and therefore, in the matter dominated era where we assume that General Relativity is recovered (i.e. $H_m \simeq 2/3t$), this gives, assuming c_m is constant,

$$\sigma_0(t) \propto \frac{t^{(1-2c_m^2)}}{1-2c_m^2}. \quad (3.16)$$

At leading order (for small sound speed), it predicts a linear behavior in t for σ_0 . Using this property, as well as the dynamics of H_m and ρ_m , in (3.15), we obtain the following expressions for the eigenvalues of \mathbf{L} at the leading order in H_m (i.e. small t)

$$\lambda_{\mathbf{L}_-} \simeq B_\zeta + \frac{3(1+\alpha_H + (1+\alpha_T)\beta_1)^2}{2M^2(1+\alpha_B)^2}, \quad \lambda_{\mathbf{L}_+} \simeq -\frac{H_m^2}{M^2} - \frac{3(1+\alpha_H + (1+\alpha_T)\beta_1)^2}{2M^2(1+\alpha_B)^2}. \quad (3.17)$$

Hence, $\lambda_{\mathbf{L}_+}$ in the above expression is always negative, instead the requirement that $\lambda_{\mathbf{L}_-}$ is negative leads to additional conditions. Applying to $\lambda_{\mathbf{K}_1}$ the same considerations that led from (3.15) to (3.17), we obtain

$$\lambda_{\mathbf{K}_1} \simeq A_\zeta + \frac{9\beta_1}{M^2(1+\alpha_B)^2}. \quad (3.18)$$

In conclusion, during the matter dominated phase of the universe, the conditions to avoid ghost and gradient instabilities read respectively

$$A_\zeta + \frac{9\beta_1}{M^2(1+\alpha_B)^2} > 0 \quad \text{and} \quad B_\zeta + \frac{3(1+\alpha_H + (1+\alpha_T)\beta_1)^2}{2M^2(1+\alpha_B)^2} < 0. \quad (3.19)$$

IV. APPLICATION TO A SIMPLE MODEL

Our aim in this section is to illustrate the different attractor and stability conditions derived so far. We propose a simple example in which these conditions turn out to be mutually compatible (which is not obvious, *a priori*).

We consider DHOST theories described by the functions

$$F_0 = c_2 X, \quad F_1 = 0, \quad F_2 = \frac{M_0^2}{2} + c_4 X^2, \quad A_3 = -8c_4 - \beta, \quad (4.1)$$

parametrized by the constant coefficients M_0 , c_2 , c_4 and β . The constant M_0 with a dimension of mass is clearly related to the Planck mass and the coefficients c_4 and β incorporate the strong coupling scale Λ_3 , which is characteristic of this kind of theories [22].

In principle the sign of c_2 is arbitrary and has not been fixed. However, as we will see below, in order for the different stability conditions (about the de Sitter solution) to hold simultaneously, one must take $c_2 > 0$. In that case $F_0 < 0$, and we will assume that the second term in F_2 is always subdominant thus guaranteeing that $F_2 > 0$.

A. Self-accelerating era

The self-accelerating solutions are identified by solving (2.24) which, in this case, leads to the fixed point

$$X_{dS} = -\sqrt{-\frac{2}{3(4c_4 + \beta)}} M_0. \quad (4.2)$$

The existence of a self-accelerating solution therefore imposes the condition $(4c_4 + \beta) < 0$. The corresponding Hubble parameter is given by (2.22)

$$H_{dS}^2 = -\sqrt{\frac{2}{3}} \frac{c_2 \sqrt{-(4c_4 + \beta)}}{M_0(8c_4 + 3\beta)}, \quad (4.3)$$

which is defined only if $(8c_4 + 3\beta) < 0$. Using the expressions in equation (2.25) and (2.26), it is easy to verify that this solution is an attractor since

$$\mathcal{B}/\mathcal{A} = 3H_{dS} > 0. \quad (4.4)$$

Concerning the stability with respect to linear cosmological perturbations, we find that

$$\text{Sign}(A_\zeta) = \text{Sign}[-c_2(4c_4 + \beta)], \quad \text{Sign}(B_\zeta) = \text{Sign}\left[-\frac{c_2 c_4(8c_4 + \beta)}{8c_4 + 3\beta}\right]. \quad (4.5)$$

The condition $A_\zeta > 0$ is automatically satisfied, while $B_\zeta < 0$ implies $[c_4(8c_4 + \beta)] < 0$, thus justifying the choice of $c_2 > 0$ (since we also have the condition $(8c_4 + \beta) < 0$).

To summarize, at this stage, we have obtained the conditions $c_2 > 0$, and

- If $\beta > 0$, then $c_4 < 0$;
- If $\beta < 0$, then $(4c_4 + \beta) < 0$, and c_4 can be either positive or negative depending on the condition $[c_4(8c_4 + \beta)] < 0$.

B. Matter dominated era and scaling solutions

In this section we focus on solutions which admit a matter dominated era before the de Sitter solution. This situation has been analysed in [23] in the context of covariant Galileons theories, where it has been shown that ‘scaling solutions’ play an important role, and also in [22] in an example of DHOST theories.

To understand the properties of the scaling solutions, we first rewrite the equations of motion (2.20) and (2.21) in the form³

$$\frac{dr_1}{dN} = f_1(r_1, r_2), \quad \frac{dr_2}{dN} = f_2(r_1, r_2), \quad (4.6)$$

by introducing new variables

$$r_1 = \frac{1}{\dot{\phi} H_b}, \quad r_2 = \dot{\phi}^4, \quad (4.7)$$

and using the e-folding $N = \ln b$ as a time variable. The Friedmann equation (2.19) gives a constraint equation $f_3(r_1, r_2) = 0$. It is interesting to study the fixed points, from the point of view of the system (4.6), which are determined by the conditions $f_1(r_1, r_2) = f_2(r_1, r_2) = 0$.

First of all, a simple analysis shows that we recover, as expected, that the de Sitter solution is an attractor at late times. With these variables, the de Sitter spacetime is described by the fixed point solution

$$r_{1dS} = \sqrt{\frac{-3(8c_4 + 3\beta)}{2c_2}}, \quad r_{2dS} = -\frac{2M_0^2}{3(4c_4 + \beta)}, \quad (4.8)$$

which obviously correspond to the self-accelerating solution given by equations (4.2) and (4.3). Linearising the equations of motion around this de Sitter fixed point, we find that the linear equations for $\delta r_1 \equiv r_1 - r_{1dS}$ and $\delta r_2 \equiv r_2 - r_{2dS}$ are given by

$$\frac{d\delta r_1}{dN} = -3\delta r_1, \quad (4.9)$$

while the equation for δr_2 depends only on δr_1 . Thus the de Sitter fixed point is stable in this theory, which is consistent with the de Sitter stability condition (4.4).

³ The analysis is simplified by working with the scale factor b where all equations of motion are second order. Furthermore in these scaling solutions, $H = H_b$.

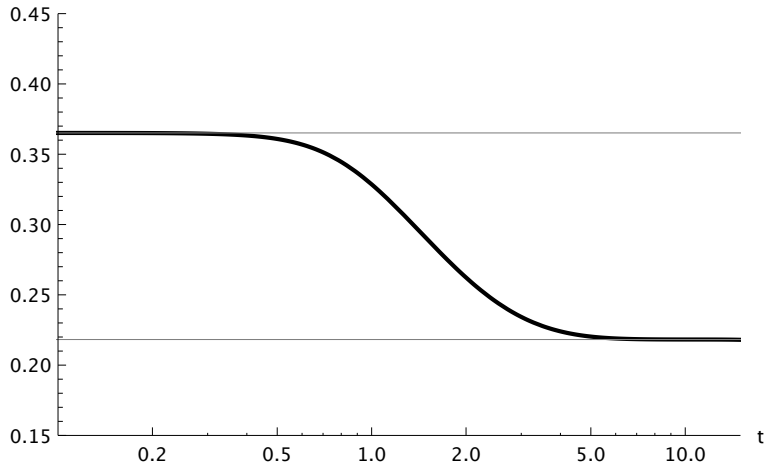


FIG. 1: $H\dot{\phi}$ as a function of time for the model with $c_2 = 1$, $c_4 = 1/2$ and $\beta = -6$. One can see the transition from the matter-dominated scaling solution ($\xi_M = \sqrt{2/15}$) to the dS solution ($\xi_{dS} = H_{dS}\sqrt{-X_{dS}} = 1/\sqrt{21}$).

Now, let us see whether there exists another fixed point (at early time) which would correspond to a matter dominated era. In the case where matter has a constant equation of state $w_m = P_m/\rho_m$, we show that there is, indeed, a new fixed point solution given by

$$r_{1m} = \sqrt{-\frac{3[16c_4 + 3(1 - w_m)\beta]}{4c_2}}, \quad r_{2m} = 0. \quad (4.10)$$

By linearising the equations of motion (4.6) about this fixed point, we find that the linear equations for the perturbations $\delta r_1 \equiv r_1 - r_{1m}$ and $\delta r_2 \equiv r_2 - r_{2m}$ are

$$\frac{d\delta r_1}{dN} = -\frac{3}{2}(3 + w_m)\delta r_1, \quad \frac{d\delta r_2}{dN} = 6(1 + w_m)\delta r_2. \quad (4.11)$$

Hence, we see immediately that, for $w_m > -1$, this fixed point is a saddle point in which the branch r_2 is unstable. Therefore, given any initial conditions, the solution of the dynamical system approaches first the point (4.10) at early times, and then it approaches the de Sitter fixed point (4.8) at late times. This is the scaling solution, and is illustrated in Fig. 1 and Fig. 2 where we also show the transition between the saddle point and the de Sitter fixed point.

It is interesting to propose an alternative way to recover the first equation of (4.10). We will use this analysis in section IV C to compute the effective equation of state of dark energy. Following the definition of r_1 in (4.7), we set

$$\dot{\phi} = \frac{\xi}{H_b}, \quad \Rightarrow \quad \ddot{\phi} = \frac{3}{2}(1 + w_m)\xi, \quad (4.12)$$

where ξ is a constant that we want to determine. Notice that we have used $H_b = 2/[3(1 + w_m)t]$ in the previous equation, as it should be at early times (large H_b), hence $\dot{\phi}$ is small whereas $\ddot{\phi}$ is constant. In order to find ξ , we substitute the equation (4.12) into the equation (2.21) and consider that H_b is large. One then finds that the dominant term in the equation (which should vanish) is proportional to the combination

$$4c_2 + 3[16c_4 + 3(1 - w_m)\beta]\xi^2.$$

Hence, the scaling solutions have

$$\xi^2 = -\frac{4c_2}{3[16c_4 + 3(1 - w_m)\beta]}, \quad (4.13)$$

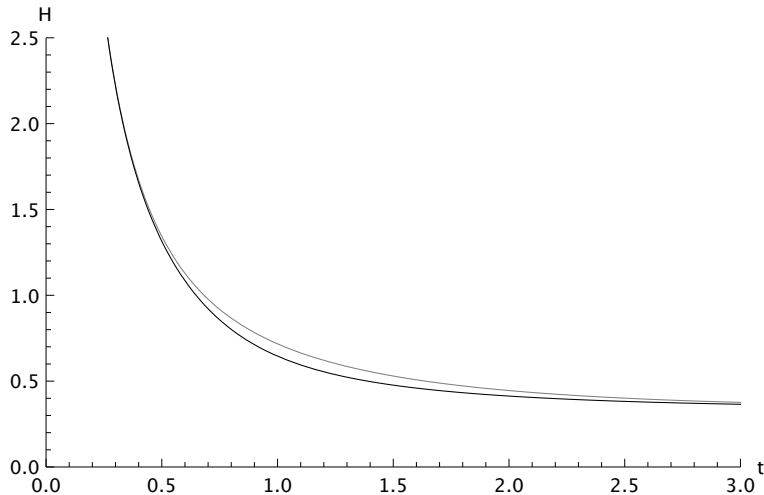


FIG. 2: The Hubble parameters H (black) and H_b (gray) as functions of time for the model with $c_2 = 1$, $c_4 = 1/2$ and $\beta = -6$. The two Hubble parameters coincide in the matter dominated regime and in the de Sitter limit, but they differ during the transition between the two regimes.

thus confirming the result of (4.10). In the matter era, this reduces to

$$\xi_M^2 = -\frac{4c_2}{3(16c_4 + 3\beta)}, \quad (4.14)$$

implying that $(16c_4 + 3\beta) < 0$.

To finish, let us discuss the stability of the scaling solution with respect to cosmological perturbations, in the matter dominated era (early times). For that purpose, we substitute this scaling solution in the conditions (3.19), under the same assumption that H_b is large, and we find that the second condition is always satisfied since, at early times, the dominant term is $-3/2$. On the other hand, the first condition gives at the leading order the stability condition $(32c_4 + 3\beta) < 0$.

Putting all together the conditions we have derived so far, we obtain the final constraint on the parameters of the model

$$c_2 > 0, \quad c_4 > 0, \quad \beta < -\frac{32}{3}c_4. \quad (4.15)$$

C. Effective equation of state for dark energy during matter domination

It is instructive to calculate the effective equation of state for dark energy, which is subdominant in the matter dominated era. To do so, we rewrite the two Friedmann equations, in the physical frame, in the form

$$3M_0^2 H^2 = \rho_m + \rho_{\text{de}}, \quad M_0^2 (2\dot{H} + 3H^2) = -P_m - P_{\text{de}}, \quad (4.16)$$

where ρ_{de} and P_{de} are the effective energy density and pressure due to the modification of gravity. In the matter dominated regime, we have $P_m = 0$. Using the two Friedmann equations (2.18) and (2.19), one finds

$$\rho_{\text{de}} = -\xi_M^2 [c_2 + 30c_4\xi_M^2] H^{-2}, \quad (4.17)$$

and

$$P_{\text{de}} = -\xi_M^2 \left[c_2 - 3 \left(8c_4 - \frac{9}{4}\beta \right) \xi_M^2 \right] H^{-2}. \quad (4.18)$$

Hence, the effective equation of state becomes

$$w_{\text{de}} = \frac{c_2 - 3 \left(8c_4 - \frac{9}{4}\beta \right) \xi_M^2}{c_2 + 30c_4\xi_M^2}, \quad (4.19)$$

and substituting (4.14) yields

$$w_{\text{de}} = -2. \quad (4.20)$$

This is expected from the scaling $\rho_{\text{de}} \propto P_{\text{de}} \propto H^{-2}$. If $\beta = 0$, which corresponds to the case of Horndeski and Beyond Horndeski theories, one recovers the result obtained in [23], which was shown to be problematic in explaining the observational data [39]. The models we have considered should however be seen as only illustrative because of their analytical simplicity, and one could extend them to overcome this difficulty.

V. CONCLUSION

In this work, we have studied cosmological solutions and their stability properties in Degenerate Higher Order Scalar Tensor (DHOST) theories. Even though we have computed and presented the cosmological (background) equations for any quadratic DHOST theory, we have mainly studied cosmological properties of isokinetic DHOST theories for which gravity waves propagate at the light speed. Isokinetic theories are characterized by $A_1 = A_2 = 0$ in the action (2.1).

We started with the study of homogenous cosmological solutions. In this simplified framework, we see explicitly how degeneracy conditions enable us to recast higher order equations of motion (and then a higher order Lagrangian) into a second order system (associated to a Lagrangian with no higher derivatives) by a change of variables. This change of variables corresponds in fact to a change of frame by a disformal transformation of the metric in the fully covariant theory that transforms the DHOST action into a Horndeski action. We have derived the two Friedmann equations in this “Horndeski frame” (where matter is not coupled minimally to the metric) from which we have easily identified the conditions for having self-accelerating de Sitter solutions, and also the conditions for these solutions to be attractors. Then, we have studied the stability of linear perturbations around these solutions in the language of the effective description of DHOST theories [26].

In the last section, we have applied all these results to a simple class of theories, showing that de Sitter attractor conditions, no ghost and no gradient instabilities conditions – both in the self-accelerating era and in the matter dominated era – can be compatible. For the latter conditions, we have considered the case where matter is described in terms of a k-essence Lagrangian. In this paper we have extended the work initiated in [23] (for generalized galileons) and in [22] (for a simple class of DHOST theories) and found the existence of stable scaling solutions which interpolate between a matter dominated era ($w_m = 0$) and a self-accelerating one. For these solutions, we computed the effective equation of state of dark energy in the matter dominated era.

Besides the constraint on the anomalous speed of gravity from GW170817 (i.e. $A_1 = A_2 = 0$), it was claimed very recently that the function A_3 should also vanish in order to prevent a catastrophic decay of gravitational waves into dark energy, an effect which would make GWs unobservable [40]. This effect would further restrict the available class of DHOST models. However, a way out of

all these constraints is to consider that the effective field theory described by DHOST is valid on cosmological scales but not necessarily on the much higher energy scales probed by LIGO/Virgo, as pointed out in [41]. Moreover, a recent paper [42] suggested an interesting possibility to get around the above constraints by exploiting the dynamics of the scalar field itself, although this proposal ultimately fails when including the effect of large scale inhomogeneities.

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Appendix A: Non-minimal coupling of matter in the Horndeski frame

Let us start with matter in the physical frame. As a direct consequence of the definition of the energy-momentum tensor

$$T^{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta S_m}{\delta g_{\mu\nu}}, \quad (\text{A1})$$

variation of the homogeneous matter action with respect to N and to a defines the matter energy density and pressure, respectively, according to

$$\delta S_m = -a^3 \rho_m \delta N + P_m \delta(a^3). \quad (\text{A2})$$

We now wish to derive the analog of (A2) in the Horndeski frame. The simplest way to proceed is to use the explicit relation between the two scale factors a and b , see (2.10), to get

$$\frac{\delta a}{a} = -2X\lambda_X \frac{\delta N}{N} + \frac{\delta b}{b}. \quad (\text{A3})$$

Substituting into (A2), one finds

$$\delta S_m = b^3 \Lambda^3 \left[-(\rho_m + 6X\lambda_X P_m) \delta N + 3P_m \frac{\delta b}{b} \right]. \quad (\text{A4})$$

As a consequence, the energy density and pressure in the Horndeski frame are defined by

$$\tilde{\rho}_m = (1 + 6X\lambda_X w_m) \Lambda^3 \rho_m, \quad \tilde{P}_m = \Lambda^3 P_m. \quad (\text{A5})$$

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